Ludwigs-Maximilians-Universität München
Fakultät für Physik
Statistical Methods - An Introduction
Problem Sheet 4 (SS12)

To be submitted at May, 14th, 2012 to jochen.weller@usm.lmu.de.

Type Ia Supernovae Magnitude Statistics

Type Ia Supernovae are among the brightest objects in the Universe. They are all almost the same brightness, with differences that can be standardized to less than 10 percent. The accelerating cosmic expansion first inferred from observations of these objects (Riess et al. 1998; Perlmutter et al. 1999) indicates unexpected gravitational physics, frequently attributed to the dominating presence of a *dark energy* with negative pressure.
We will now use a SN Ia data sample compiled by Riess et al. 2004 to estimate the matter content of the Universe.
After downloading the supernovae dataset `sn_data_riess.dat` on lecture webpage, we need to:

a) Read data points using the `scan` command: we need columns 2,3,4 which correspond to redshift z, magnitude m and error on magnitude σ (be careful to skip the header). Define a table called `data` with these three columns: it will be useful later.

b) Calculate the theoretical magnitude, assuming a flat Universe with $H_0 = 72.0$ $[kms^{-1}Mpc^{-1}]$, $c = 300000$ $[kms^{-1}]$.
The luminosity distance d_L which is given by

$$d_L = c(1+z)\int_0^z \frac{dz}{H(z)} \ ,$$

is implemented in the `dL.R` code, which can be dowloaded from lecture webpage.
Define then the magnitude m as a function of z, Ω_m, M, remembering its definition

$$m = M + 5log_{10}(H_0 \cdot d_L) \ .$$

[Note that this is obtained from

$$m = M_{int} + 5log_{10}(d_L) + 25 \ ,$$

where

$$M = M_{int} - 5log_{10}(H_0) + 25$$

and M_{int} is the intrinsic magnitude.]

c) Define χ^2 as a function of Ω_m and M, according to its definition

$$\chi^2 = \sum_{i=1}^{N} \frac{(m_i - m_{th}(z_i))^2}{\sigma_i^2} \ ,$$

where N is the number of data points, m_i is magnitude data, m_{th} is theoretical magnitude, σ_i is the error on data.

d) Find the minimum of χ^2 (hint: avoid calculating the derivative and use `for` loops instead) and print out also the minimum values for the parameters Ω_m and M. Let vary Ω_m in $[0.0, 1.0]$ with step 0.05 and M in $[15.5, 16.5]$ with step 0.01.

e) Calculate the posterior probability as a function of Ω_m and M, according to its definition

$$p = e^{-\frac{1}{2}(\chi^2 - \chi^2_{min})} \ ,$$

where χ^2_{min} is the value calculated in the previous step.

f) Plot Ω_m-M contour plot (hint: see Lecture3b).

g) Marginalize over M in the range $[13.0, 18.0]$ (hint: see Lecture3c).

h) Plot the 1-d marginalized likelihood as a function of Ω_m. 10 Points